

On the finite temperature $\lambda\varphi^4$ and Gross-Neveu models. Is there a first order phase transition in $(\lambda\varphi^4)_{D=3}$?

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Abstract

We study the behavior of two different models at finite temperature in a D -dimensional space-time. The first one is the $\lambda\varphi^4$ model and the second one is the Gross-Neveu model. Using the one-loop approximation we show that in the $\lambda\varphi^4$ model the thermal mass increase with the temperature while the thermal coupling constant decrease with the temperature. Using this facts we establish that in the $(\lambda\varphi^4)_{D=3}$ model there is a temperature β_\star^{-1} above which the system can develop a first order phase transition, where the origin corresponds to a metastable vacuum. In the massless Gross-Neveu model, we demonstrate that for $D = 3$ the thermal correction to the coupling constant is zero. For $D \neq 3$ our results are inconclusive.

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1 Introduction

In the last years, there has been much interest in the nature of the electroweak phase transition. The high temperature effective potential in the standard and in the $(\lambda\varphi^4)_{D=4}$ models have been calculated by many authors, where the contribution from multiloops diagrams has been taking into account. Several authors have pointed out the importance to know whether in $(\lambda\varphi^4)_{D=4}$ model the phase transition is of first or second order [1]. Our interest in these issues was stimulated by some results of Ford and Svaiter concerning the thermal dependence of the mass and coupling constant in $(\lambda\varphi^4)_{D=4}$ model defined in a non-simple connected spacetime [2]. In the aforementioned paper these authors studied a neutral scalar field in a $D = 4$ dimensional spacetime using the one-loop effective potential. The cases of trivial and non-trivial topology of the spacelike sections and finite temperature were discussed. The temperature and topological dependent renormalized mass and coupling constant were derived using the Speer and Bollini, Giambiagi and Domingues analytic regularization [3] and a modified minimal subtraction renormalization procedure [4]. In addition they have also discussed the possibility of vanishing the renormalized coupling constant in this model, as well as the limits of validity of the one-loop approximation. Some calculations studying such kind of problems was given recently by Elizalde and Kirsten and also Villareal [5]. This last author improved the precedent results studying the two-loops corrections to the effective potential for scalar fields defined in a spacetime with non-trivial topology of the spacelike sections.

The two goals of this paper are the following. The first one is to extend the discussion of

the massive self-interacting $\lambda\varphi^4$ model to an arbitrary D -dimensional spacetime, assuming trivial topology of the spacelike sections and to analyze temperature effects in a model with asymptotic freedom. The second one is to discuss the existence of a first order phase transition in the massive $(\lambda\varphi^4)_{D<4}$ model.

Besides Yang-Mills theories in $D = 4$, the other known perturbative renormalizable asymptotically free theories with fermions are the Nambu-Jona-Lasinio and the Gross-Neveu models [6]. In the latter, a N component fermion field with a quartic self-interaction is assumed. The model is perturbatively renormalizable for $D = 2$ and develops asymptotic freedom.

Working in a generic D -dimensional spacetime, we first calculate the one-loop corrections to the renormalized mass and coupling constant in the $\lambda\varphi^4$ model. We obtained that the thermal mass increase and the thermal coupling constant decrease with the temperature. Still using the one-loop approximation, the thermal correction to the renormalized coupling constant in the Gross-Neveu model is obtained. We demonstrate that in the case $D = 3$ the thermal correction to the coupling constant is zero. For $D \neq 3$ our results are inconclusive.

In many papers studying second order phase transition in the $\lambda\varphi^4$ model the temperature dependence of the coupling constant is neglected. This approach is reasonable since the variation of the mass with the temperature is the most important fact for a critical phenomena. In this case, it is sufficient to consider the renormalized coupling constant as constant and the thermal mass drives the second order phase transition [7]. In this paper we will examine the existence of a first order phase transition in the $\lambda\varphi^4$ model taking into account the thermal dependence of the coupling

constant. Note that we will not deal with the system behavior in the neighborhood of a second order phase transition since we assume that the tree level mass squared m^2 is positive. This fact prevents the one-loop approximation to break down at low temperatures since there is no infrared divergences associated with vanishing masses. The result of our analysis can be summarized as follows: for $D < 4$, there is a temperature β_\star^{-1} where the effective coupling constant vanishes. For temperatures $\beta^{-1} > \beta_\star^{-1}$, the renormalized coupling constant becomes negative and the system may suffer a first order phase transition. The effects of the radiative corrections is toward the direction of breaking a symmetry. Compare with the electroweak first order phase transition [8]. We should note that at $\beta^{-1} = \beta_\star^{-1}$ the system is still in an interacting phase. For $D < 4$, there is a temperature where only the effective coupling constant ($\lambda(\beta) = \lambda - \lambda^2 f(\beta)$) vanishes. All the higher 2n-points correlation functions do not vanish, therefore the model is not gaussian at the temperature β_\star^{-1} . This is an important point that was stressed by Weldon [9].

The study of the dependence of the coupling constant with the temperature in QFT is well known in the literature. Many authors have studied such dependence in the $\lambda\varphi^4$ model and also in a abelian model like QED [10]. Instead of using perturbative arguments, the use of the renormalization group equations allowed the investigation on the mass and coupling constant thermal dependence. Such program was implemented by Fujimoto, Ideura, Nakano and Yoneyama [11]. These authors obtained results similar to ours in the $\lambda\varphi^4$ model. The behavior of the mass and coupling constant with the temperature are opposite, i.e. the renormalized mass increases if the system temperature increase as where the coupling constant decreases. If we assume that the one-

loop approximation provides trustable results we have the following situation: for temperatures above β_{\star}^{-1} the renormalized coupling constant becomes negative. This behavior of the effective coupling constant is related to the fact that the model is non-asymptotically free. The growth of the renormalized coupling constant at large momenta is translated in our case to the temperature growth (in modulus) of this quantity. In $D = 3$ for temperatures $\beta^{-1} > \beta_{\star}^{-1}$ the system develop a first order phase transition where the origin is a metastable vacuum.

In this paper we address only the one-loop approximation. It is not unreasonable to believe that our conclusions in the $\lambda\varphi^4$ model may be limited to this approximation. In fact, the behavior of the thermal correction to the coupling constant changes in the two-loops approximation. It was been shown by Funakubo and Sakamoto [10] that only for *low* temperatures the behavior of the thermal coupling constant remains the same as the obtained in the one-loop approximation. For high temperatures ($\beta^{-1} \gg m$) the behavior is opposite i.e., the thermal correction is positive. Nevertheless this fact does not exclude the possibility of a first order phase transition at low temperatures in $(\lambda\varphi^4)_{D=3}$. A more detailed discussion will appear in a forthcoming paper.

The paper is organized as follows. In section II we sketch the formalism of the effective potential. In section III, the massive self-interacting $\lambda\varphi^4$ model is analised. In section IV we repeat the calculations in the Gross-Neveu model. Conclusions are given in section V . In this paper we use $\frac{\hbar}{2\pi} = c = 1$.

2 The effective action and the effective potential at zero temperature.

In this chapter we will review briefly the basic features of the effective potential associated with a real massive self-interacting scalar field at zero temperature. Although the formalism of this section may be found in standard textbooks, we recall here its main results for completeness. Let us suppose a real massive scalar field $\varphi(x)$ with the usual $\lambda\varphi^4(x)$ self-interaction, defined in a static spacetime. Since the manifold is static, there is a global timelike Killing vector field orthogonal to the spacelike sections. Due to this fact, energy and thermal equilibrium have a precise meaning. For the sake of simplicity, let us suppose that the manifold is flat. In the path integral approach, the basic object is the generating functional,

$$Z[J] = \langle 0, out | 0, in \rangle = \int \mathcal{D}[\varphi] \exp\{i[S[\varphi] + \int d^4x J(x)\varphi(x)]\} \quad (1)$$

where $\mathcal{D}[\varphi]$ is the functional measure and $S[\varphi]$ is the classical action associated with the scalar field. The quantity $Z[J]$ gives the transition amplitude from the initial vacuum $|0, in \rangle$ to the final vacuum $|0, out \rangle$ in the presence of some source $J(x)$, which is zero outside some interval $[-T, T]$ and inside this interval was switched adiabatically on and off. Since we are interested in the connected part of the time ordered products of the fields, we take the connected generating functional $W[J]$, as usual. This quantity is defined in terms of the vacuum persistent amplitude

by

$$e^{iW[J]} = \langle 0, out | 0, in \rangle. \quad (2)$$

The connected n -point function $G_c(x_1, x_2, \dots, x_n)$ is defined by

$$G_c(x_1, x_2, \dots, x_n) = \frac{\delta^n W[J]}{\delta J(x_1) \dots \delta J(x_n)} \Big|_{J=0}. \quad (3)$$

Expanding $W[J]$ in a functional Taylor series, the n -order coefficient of this series will be the sum of all connected Feynman diagrams with n external legs, i.e. the connected Green's functions defined by eq.(3). Then

$$W[J] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \dots d^4x_n G_c^{(n)}(x_1, x_2, \dots, x_n) J(x_1) J(x_2) \dots J(x_n). \quad (4)$$

The classical field φ_0 is given by the normalized vacuum expectation value of the field

$$\varphi_0(x) = \frac{\delta W}{\delta J(x)} = \frac{\langle 0, out | \varphi(x) | 0, in \rangle_J}{\langle 0, out | 0, in \rangle_J}, \quad (5)$$

and the effective action $\Gamma[\varphi_0]$ is obtained by performing a functional Legendre transformation

$$\Gamma[\varphi_0] = W[J] - \int d^4x J(x) \varphi_0(x). \quad (6)$$

Using the functional chain rule and the definition of φ_0 given by eq.(5) we have

$$\frac{\delta \Gamma[\varphi_0]}{\delta \varphi_0} = -J(x). \quad (7)$$

Just as $W[J]$ generates the connected Green's functions by means of a functional Taylor expansion, the effective action can be represented as a functional power series around the value $\varphi_0 = 0$,

where the coefficients are just the proper n -point functions $\Gamma^{(n)}(x_1, x_2, \dots, x_n)$ i.e.,

$$\Gamma[\varphi_0] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 d^4x_2 \dots d^4x_n \Gamma^{(n)}(x_1, x_2, \dots, x_n) \varphi_0(x_1) \varphi_0(x_2) \dots \varphi_0(x_n). \quad (8)$$

The coefficients of the above functional expansion are the connected 1 particle irreducible diagrams (1PI). Actually, $\Gamma^{(n)}(x_1, x_2, \dots, x_n)$ is the sum of all 1PI Feynman diagrams with n external legs. Writing the effective action in powers of momentum (around the point where all external momenta vanish) we have

$$\Gamma[\varphi_0] = \int d^4x \left(-V(\varphi_0) + \frac{1}{2} (\partial_\mu \varphi)^2 Z[\varphi_0] + \dots \right). \quad (9)$$

The term $V(\varphi_0)$ is called the effective potential[12][13]. To express $V(\varphi_0)$ in terms of the 1PI Green's functions, let us write $\Gamma^{(n)}(x_1, x_2, \dots, x_n)$ in the momentum space:

$$\Gamma^{(n)}(x_1, x_2, \dots, x_n) = \frac{1}{(2\pi)^n} \int d^4k_1 d^4k_2 \dots d^4k_n (2\pi)^4 \delta(k_1 + k_2 + \dots k_n) e^{i(k_1 x_1 + \dots k_n x_n)} \tilde{\Gamma}^{(n)}(x_1, x_2, \dots, x_n). \quad (10)$$

Assuming that the model is translationally invariant, i.e. φ_0 is constant over the manifold, we have

$$\Gamma[\varphi_0] = \int d^4x \sum_{n=1}^{\infty} \frac{1}{n!} \left(\tilde{\Gamma}^{(n)}(0, 0, \dots) (\varphi_0)^n + \dots \right). \quad (11)$$

If we compare eq.(9) with eq.(11) we obtain that

$$V(\varphi_0) = - \sum_n \frac{1}{n!} \tilde{\Gamma}^{(n)}(0, 0, \dots) (\varphi_0)^n, \quad (12)$$

then $\frac{d^n V}{d\varphi_0^n}$ is the sum of the all 1PI diagrams carrying zero external momenta. Assuming that the fields are in equilibrium with a thermal reservoir at temperature β^{-1} , in the Euclidean time

formalism, the effective potential $V(\beta, \varphi_0)$ can be identified with the free energy density and can be calculated by imposing periodic (antiperiodic) boundary conditions on the bosonic (fermionic) fields.

In the next section using the effective potential we will perform the one-loop renormalization of the $\lambda\varphi^4$ assuming that the system is in equilibrium with a thermal reservoir at temperature β^{-1} . Since we are interested to make a paralel with the tricritical phenomena where in the tree level approximation with $V(\varphi) = m^2\varphi^2 + \lambda\varphi^4 + \sigma\varphi^6$ predicts the existence of a first order phase transition if we allow the coefficient of the quartic term to be negative, we will evaluate the effective potential in a very unusual way. Instead of summing the series obtaining a log expression, and regularizing the model by introducing an ultraviolet cut-off in the Euclidean momenta, we prefer to use the principle of analytic extension in each term of the series. The advantage of this method lies in the fact that the dependence of mass and coupling constant with the temperature appear in a very straightforward way as well as the paralel with the tricritical phenomena.

3 The one-loop effective potential in the $\lambda\varphi^4$ model at zero and finite temperature.

Let us assume the following Lagrange density associated with a massive neutral scalar field:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi_u)^2 - \frac{1}{2}m_0^2\varphi_u^2 - \frac{\lambda_0}{4!}\varphi_u^4 \quad (13)$$

where $\varphi_u(x)$ is the unrenormalized field and m_0 and λ_0 are the bare mass and bare coupling constant respectively. We may rewrite the Lagrange density as the usual form where the counterterms will appear explicitly. Defining the quantities

$$\varphi_u(x) = (1 + \delta Z)^{\frac{1}{2}} \varphi(x) \quad (14)$$

$$m_0^2 = (m^2 + \delta m^2)(1 + \delta Z)^{-1} \quad (15)$$

$$\lambda_0 = (\lambda + \delta \lambda)(1 + \delta Z)^{-2}, \quad (16)$$

and substituting eq.(14),(15) and (16) in eq.(13) we have

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2 \varphi^2 - \frac{\lambda}{4!} \varphi^4 + \frac{1}{2} \delta Z (\partial_\mu \varphi)^2 - \frac{1}{2} \delta m^2 \varphi^2 - \frac{1}{4!} \delta \lambda \varphi^4, \quad (17)$$

where δZ , δm^2 , and $\delta \lambda$ are the wave function, mass and coupling constant counterterms of the model. After the Wick rotation, in the one-loop approximation, the effective potential is given by [13]:

$$V(\varphi_0) = V_I(\varphi_0) + V_{II}(\varphi_0) \quad (18)$$

where,

$$V_I(\varphi_0) = \frac{1}{2}m^2 \varphi_0^2 + \frac{\lambda}{4!} \varphi_0^4 - \frac{1}{2} \delta m^2 \varphi_0^2 - \frac{1}{4!} \delta \lambda \varphi_0^4, \quad (19)$$

and

$$V_{II}(\varphi_0) = \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{2s} \left(\frac{1}{2} \lambda \varphi_0^2 \right)^s \int \frac{d^D q}{(2\pi)^D} \frac{1}{(\omega^2 + \vec{q}^2 + m^2)^s}. \quad (20)$$

Before continuing, we would like to discuss one important point. Performing analytic or dimensional regularization, we must introduce a mass parameter μ , in terms of which dimensional

analysis gives to the field a dimension $[\varphi] = \mu^{1/2(D-2)}$ and to the coupling constant a dimension $[\lambda] = \mu^{4-D}$. Mass has dimension of inverse length, i.e. $[\mu] = [m] = L^{-1}$, and the effective potential (the energy density per unit volume) has dimension of L^{-D} .

It is not difficult to extend the results given by eqs.(19) and (20) to finite temperature states. After a Wick rotation, the functional integral runs over the fields that satisfy periodic boundary conditions in Euclidean time. The effective action can be defined as in the zero temperature case by a functional Legendre transformation. Regularization and renormalization procedures follow the same steps as in the zero temperature case. Although the counterterms introduced at finite temperature are the same as in the zero temperature case, the finite part of the physical parameters are temperature dependent. In this situation, since the sign of the thermal correction to the coupling constant is negative, the possibility of vanishing the renormalized coupling constant appears.

To study temperature effects we perform as usual the following replacement in the Euclidean region:

$$\int \frac{d\omega}{2\pi} \rightarrow \frac{1}{\beta} \sum_n \quad (21)$$

and

$$\omega \rightarrow \frac{2\pi n}{\beta} \quad (22)$$

where $\omega_n = \frac{2\pi n}{\beta}$ are the Matsubara frequencies. Defining the dimensionless quantities:

$$c^2 = \frac{m^2}{4\pi^2\mu^2} \quad (23)$$

and

$$(\beta\mu)^2 = a^{-1}, \quad (24)$$

the Born terms plus one-loop terms contributing to the effective potential give,

$$V(\beta, \varphi_0) = V_I(\varphi_0) + V_{II}(\beta, \varphi_0)$$

where,

$$V_I(\beta, \varphi_0) = \frac{1}{2}m^2\varphi_0^2 + \frac{\lambda}{4!}\varphi_0^4 - \frac{1}{2}\delta m^2\varphi_0^2 - \frac{1}{4!}\delta\lambda\varphi_0^4, \quad (25)$$

and

$$V_{II}(\beta, \varphi_0) = \frac{1}{\beta} \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{2s} \left(\frac{\lambda}{8\pi^2} \right)^s \left(\frac{\varphi_0}{\mu} \right)^{2s} \int \frac{d^d q}{(2\pi)^d} A_1^{M^2}(s, a). \quad (26)$$

The function

$$A_N^{c^2}(s, a_1, a_2, \dots, a_N) = \sum_{n_1, n_2, \dots, n_N = -\infty}^{\infty} (a_1 n_1^2 + a_2 n_2^2 + \dots + a_N n_N^2 + c^2)^{-s} \quad (27)$$

is the inhomogeneous Epstein zeta function[14], and finally

$$M^2 = \frac{1}{4\pi^2\mu^2}(\vec{q}^2) + c^2.$$

Note that the mass parameter μ introduced in eqs.(23) and (24) will be used from now on, since we must have dimensionless functions when working with analytic extensions.

Let us define the modified inhomogeneous Epstein zeta function as

$$E_N^{c^2}(s, a_1, a_2, \dots, a_N) = \sum_{n_1, n_2, \dots, n_N = 1}^{\infty} (a_1 n_1^2 + \dots + a_N n_N^2 + c^2)^{-s}. \quad (28)$$

Defining the new coupling constant and a new vacuum expectation value of the field ϕ (dimensionless for $D = 4$),

$$g = \frac{\lambda}{8\pi^2} \quad (29)$$

$$\frac{\varphi_0}{\mu} = \phi \quad (30)$$

$$k^i = \frac{q^i}{2\pi\mu} \quad (31)$$

we rewrite eq.(26) without use the definition of the inhomogeneous Epstein zeta function as,

$$V_{II}(\beta, \phi) = \mu^D \sqrt{a} \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{2s} g^s \phi^{2s} \sum_{n=-\infty}^{\infty} \int d^d k \frac{1}{(an^2 + c^2 + \vec{k}^2)^s}. \quad (32)$$

To regularize the model we will use a mix between dimensional and zeta function analytic regularizations. Let us first use dimensional regularization[15]. Using the well known result,

$$\int \frac{d^d k}{(k^2 + a^2)^s} = \frac{\pi^{\frac{d}{2}}}{\Gamma(s)} \Gamma(s - \frac{d}{2}) \frac{1}{a^{2s-d}}, \quad (33)$$

eq. (32) becomes

$$V_{II}(\beta, \phi) = \mu^D \sqrt{a} \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{2s} g^s \phi^{2s} \frac{\pi^{\frac{d}{2}}}{\Gamma(s)} \Gamma(s - \frac{d}{2}) \sum_{n=-\infty}^{\infty} \frac{1}{(an^2 + c^2)^{s-\frac{d}{2}}}. \quad (34)$$

Defining,

$$f(D, s) = f(d+1, s) = \frac{(-1)^{s+1}}{2s} \pi^{\frac{d}{2}} \Gamma(s - \frac{d}{2}) \frac{1}{\Gamma(s)} \quad (35)$$

and substituting eqs.(27) and (35) in eq.(34) we obtain,

$$V_{II}(\beta, \phi) = \mu^D \sqrt{a} \sum_{s=1}^{\infty} f(D, s) g^s \phi^{2s} A_1^{c^2}(s - \frac{d}{2}, a). \quad (36)$$

As we will soon see, the terms $s \leq \frac{D}{2}$ are divergent and we will regularize the one-loop effective potential using the Principle of the Analytic Extension. Let us assume that each term in the series of the one-loop effective potential $V(\beta, \phi)$ is the analytic extension of these terms, defining in the beginning in an open connected set. To render the discussion more general, let us discuss the process of the analytic continuation of the modified inhomogeneous Epstein zeta function given by eq.(28). For $Re(s) > \frac{N}{2}$, the $E_N^{c^2}(s, a_1, a_2, ..a_N)$ converges and represent an analytic function of s , so $Re(s) > \frac{N}{2}$ is the largest possible domain of the convergences of the series. This means that in eq.(36) in the case $D = 4$ only the terms $s = 1$ and $s = 2$ are divergent. The term $s = 1$ is the divergent one-loop diagram of the connected two-point function and it contributes with a quadratic divergence. The $s = 2$ term is the divergent one-loop diagram of the connected four point function, and it contributes to the effective potential with a logarithmic divergence. Using a Mellin transform it is possible to find the analytic extension of the modified inhomogeneous Epstein zeta function. After some calculations using Kirsten's results [16], we have:

$$V_{II}(\beta, \phi) = \mu^D \sum_{s=1}^{\infty} f(D, s) g^s \phi^{2s} \sqrt{\pi} \left(\frac{m}{2\pi\mu} \right)^{D-2s} \frac{1}{\Gamma(s - \frac{D}{2})} \left(\Gamma(s - \frac{D}{2}) + 4 \sum_{n=1}^{\infty} \left(\frac{mn\beta}{2} \right)^{s-\frac{D}{2}} K_{\frac{D}{2}-s}(mn\beta) \right) \quad (37)$$

where $K_{\mu}(z)$ is the Kelvin function [17].

It is not difficult to show that:

$$V_{II}(\beta, \phi) = \mu^D \sum_{s=1}^{\infty} g^s \phi^{2s} h(D, s) \left(\frac{1}{2^{\frac{D}{2}-s+2}} \Gamma(s - \frac{D}{2}) \left(\frac{m}{\mu} \right)^{D-2s} + \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2 \beta n} \right)^{\frac{D}{2}-s} K_{\frac{D}{2}-s}(mn\beta) \right) \quad (38)$$

where:

$$h(D, s) = \frac{1}{2^{\frac{D}{2}-s-1}} \frac{1}{\pi^{\frac{D}{2}-2s}} \frac{(-1)^{s+1}}{s} \frac{1}{\Gamma(s)}. \quad (39)$$

If we suppose that $D = 4$, the model is perturbatively renormalizable and an appropriate choice of δm^2 and $\delta \lambda$ will render the analytic extension of the terms of the series in s in the effective potential analytic functions in the neighbourhood of the poles $s = 1$ and $s = 2$ respectively.

The idea to extend the definition of an analytic function to a larger domain (analytic extension) and subtract poles was exploited by Speer, Bollini and others. In the method used by Bollini, Giambiagi and Domingues, a complex parameter s was introduced as an exponent of the denominator of the loop expressions and the integrals are well defined analytic functions of the parameters in the region $Re(s) > s_0$ for some s_0 . Performing an analytic extension of the expression for $Re(s) \leq s_0$, poles will appear in the analytic extension and the final expression becomes finite after a renormalization procedure. To find the exact form of the counterterms let us use the renormalization conditions

$$\frac{\partial^2}{\partial \phi^2} V(\beta, \phi)|_{\phi=0} = m^2 \mu^2 \quad (40)$$

and

$$\frac{\partial^4}{\partial \phi^4} V(\beta, \phi)|_{\phi=0} = \lambda \mu^4. \quad (41)$$

Since the vacuum expectation value of the field has been chosen to be constant, there is no need for wave function renormalization. Substituting eqs.(25),(38) and (39) in eqs.(40) and (41) it is possible to find the exact form of the counterterms in such a way that they cancel the polar parts of the analytic extension of the terms $s = 1$ and $s = 2$. Note that we are using a "modified" minimal

subtraction renormalization scheme where the mass and coupling constant counterterms are poles at the physical values of s . It is straightforward to show that both δm^2 and $\delta\lambda$ are temperature independent. If a model at zero temperature is renormalizable with some counterterms it is also renormalizable at finite temperature with the same counterterms. This result was obtained in all orders of perturbation theory by Kislinger and Morley [18]. In the neighbourhood of the poles $s = 1$ and $s = 2$, the regular part of the analytic extension of inhomogeneous Epstein zeta function has two contributions: one which is temperature independent and that can be absorbed by the counterterms and another that is temperature dependent and cannot be absorbed by the counterterms. It is clear that the temperature dependent mass is proportional to the regular part of the analytic extension of the inhomogeneous Epstein zeta function in the neighborhood of the pole $s = 1$. The same argument can be applied to the renormalized coupling constant. The thermal contribution to the renormalized coupling constant is proportional to the analytic extension of the inhomogeneous Epstein zeta function in the neighborhood of the pole $s = 2$. The choice of the renormalization point $\phi = 0$ implies that only the regular part in the neighborhood of the pole $s = 1$ will appear in the renormalized mass. In the next section (where massless self-interacting fermion fields are studied) we will show that all the terms of the series of the effective potential contribute to the renormalized mass and coupling constant and the sign of the thermal coupling constant cannot be computed for $D \neq 3$. From the above discussion we can write

$$-\tilde{\Gamma}^{(2)}(p=0, \beta, \lambda, m) = m^2(\beta) = m^2 + \Delta m^2(\beta) \quad (42)$$

and

$$-\tilde{\Gamma}^{(4)}(p=0, \beta, \lambda, m) = \lambda(\beta) = \lambda + \Delta\lambda(\beta), \quad (43)$$

where $m^2(\beta)$ and $\lambda(\beta)$ are respectively the temperature dependent renormalized mass squared and coupling constant. It can be directly shown that the thermal contribution to the renormalized mass squared is given by:

$$\Delta m^2(\beta) - \Delta m^2(\infty) = \frac{1}{8\pi^2} \lambda \sum_{n=1}^{\infty} \frac{m}{\beta n} K_1(mn\beta). \quad (44)$$

Using the asymptotic representation of the Bessel function $K_n(z)$ for small arguments

$$K_n(z) \cong \frac{1}{2} \Gamma(n) \left(\frac{z}{2}\right)^{-n}, \quad z \rightarrow 0 \quad n = 1, 2, \dots,$$

we obtain that at high temperatures the temperature dependent mass squared is proportional to $\lambda\beta^{-2}$ [19]. The result given by eq.(44) was also obtained by Braden [20] using Schwinger's proper time method. The same author also discussed the two-loop effective potential and the problem of overlapping divergences where the possibility of temperature dependent counterterms appears. Nevertheless these divergences must cancel as it was stressed by Kislinger and Morley [18].

Based upon the same arguments previously used, the thermal contribution to the renormalized coupling constant is given by:

$$\Delta\lambda(\beta) - \Delta\lambda(\infty) = -\frac{3}{8\pi^2} \lambda^2 \sum_{n=1}^{\infty} K_0(mn\beta). \quad (45)$$

The Bessel function $K_0(z)$ is positive and decreases for $z > 0$. Therefore let us present an interesting result: the renormalized coupling constant attains its maximum at zero temperature

$(\beta^{-1} = \infty)$ and decreases monotonically as the temperature increases. In other words, the thermal contribution to the renormalized coupling constant $\Delta\lambda(\beta) - \Delta\lambda(\infty)$ is negative, and increases in modulus with the temperature. The same result was obtained by Fujimoto, Ideura, Nakano and Yoneyama using the renormalization group equations at finite temperature [11]. Once we are discussing thermal effects, in the limit of zero temperature the thermal contribution to the mass and coupling constant must vanish $(\tilde{\Gamma}^{(2)}(p = 0, \beta = \infty, \lambda, m) = -m^2$ and $(\tilde{\Gamma}^{(4)}(p = 0, \beta = \infty, \lambda, m) = -\lambda)$. This can be easily seen from eqs.(44) and (45). Since the thermal contribution to the renormalized coupling constant is negative someone could enquire: is it possible for the renormalized coupling constant to vanish? Once $\Delta\lambda(\beta)$ is $O(\lambda^2)$ and we assume $D = 4$, it is not possible to implement such a mechanism for finite temperatures. For $D < 4$ the renormalized coupling constant is not necessarily a small quantity and it can even become a large quantity, due to its positive dimension $4 - D$ in terms of the mass parameter μ (or using the language of critical phenomena, due to its positive dimension $4 - D$ in terms of the scale $\frac{1}{a}$ where a is the lattice spacing). Therefore we conclude that in the neighbourhood of $D = 4$, the renormalized coupling constant $\lambda(\beta)$ could vanish only for very high temperatures. As we consider smaller spacetime dimensions the temperature where $\lambda(\beta)$ vanishes becomes lower and lower. For instance, for $D = 3$ we expect to find a finite temperature β_\star^{-1} such that the renormalized coupling constant vanishes.

We note that there is no discontinuity in the behavior between the cases $D = 4$ and $D < 4$ as we will see later (see eq.(49)). For $D < 4$ the model becomes superrenormalizable and only a finite number set of graphs need overall counterterms. In the one-loop approximation for $D = 4$

there are only two divergent graphs and for $D < 4$ there is only one. This result can be easily obtained by investigating eq.(38). In this equation the divergent part of the effective potential is given by $\Gamma(s - \frac{D}{2})$ and for $D < 4$ only the $s = 1$ pole will appear. In other words, for $D < 4$ there is only finite coupling constant renormalization at the one-loop approximation. The graph $s = 2$ gives a finite and negative contribution to the coupling constant. For $D \geq 4$ the renormalization of the coupling constant is obligatory (note the presence of the pole in $s = 2$). Going back to the D -dimensional case, the renormalization conditions also are given by eqs.(40) and (41). Using the renormalization conditions in eq.(38), we can find the regular part of the analytic extension which gives a finite contribution to the renormalized mass squared $\Delta m^2(D, m, \lambda, \beta)$ and coupling constant $\Delta \lambda(D, m, \lambda, \beta)$ in a D -dimensional flat spacetime. We will simplify the notation writing $\Delta m^2(\beta)$ and $\Delta \lambda(\beta)$. For even D they are given respectively by:

$$\Delta m^2(\beta) = \frac{\mu^{D-2}\lambda}{2(2\pi)^{D/2}} \left(\frac{(-1)^{\frac{D}{2}-1}}{(\frac{D}{2}-1)!} \psi\left(\frac{D}{2}\right) \left(\frac{m}{\mu}\right)^{D-2} + \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-1} K_{\frac{D}{2}-1}(mn\beta) \right) \quad (46)$$

and

$$\Delta \lambda(\beta) = -\frac{3}{2} \frac{\mu^{D-4}\lambda^2}{(2\pi)^{D/2}} \left(\frac{(-1)^{\frac{D}{2}-2}}{(\frac{D}{2}-2)!} \psi\left(\frac{D}{2}-1\right) \left(\frac{m}{\mu}\right)^{D-4} + \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta) \right) \quad (47)$$

where $\psi(s) = \frac{d}{ds} \ln \Gamma(s)$. For odd D , the first term between parenthesis in eqs.(46) and (47) must be replaced by $\Gamma(1 - \frac{D}{2}) (\frac{m}{\mu})^{D-2}$ and $\Gamma(2 - \frac{D}{2}) (\frac{m}{\mu})^{D-4}$ respectively. The first terms between parenthesis of eq.(46) and eq.(47) are temperature independent therefore it is possible to isolate the thermal contribution to the renormalized mass and coupling constant in a generic D -dimensional spacetime in the one-loop approximation. Using eq.(46) and eq.(47) we obtain the following contribution to

the thermal mass and coupling constant respectively:

$$\Delta m^2(\beta) - \Delta m^2(\infty) = \frac{\mu^{D-2}\lambda}{2(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n} \right)^{\frac{D}{2}-1} K_{\frac{D}{2}-1}(mn\beta) \quad (48)$$

and

$$\Delta\lambda(\beta) - \Delta\lambda(\infty) = -\frac{3}{2} \frac{\mu^{D-4}\lambda^2}{(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n} \right)^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta). \quad (49)$$

These are among the main results of the paper. Since $\Delta\lambda(\beta) - \Delta\lambda(\infty) < 0$ we may have a temperature β_\star^{-1} where $\lambda(\beta)$ vanish for $D < 4$. Our result is different from the Frohlich result [21] in which all the Green's functions of the theory for $D > 4$ correspond to a free field i.e. the model is gaussian at zero temperature above four spacetime dimensions. In our case, the higher $2n$ -point functions are not zero as was discussed by Weldon [9].

Before discussing a existence of a first order phase transition, we would like to point out that the investigation of the $(\lambda\varphi^4)_{D=4}$ model with a negative bare coupling constant has recently been done by Langfeld et al, where an analytic continuation of the model with positive λ to negative values was presented [22]. Although several authors claim that the renormalized coupling constant of the $\lambda\varphi^4$ model must be positive, a definitive supporting argument is still lacking. Previous investigations have been done by many authors [23]. We would like to stress that the sign of the renormalized coupling constant is not fixed by the renormalization procedure in the $(\lambda\varphi^4)_{D=4}$. Gallavoti and Rivasseau discussed examples with positive bare coupling constant where different cutoffs lead to renormalized coupling constants with different signs [24].

Going back to the discussion of a first order phase transition, let us define a dimensionless

effective potential $v = \frac{V}{\mu^D}$, as:

$$\begin{aligned}
v(\beta, \phi) &= \frac{1}{2}m^2\mu^{2-D}\phi^2 + \frac{\lambda}{4(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-1} K_{\frac{D}{2}-1}(mn\beta)\phi^2 \\
&+ \frac{\lambda}{4!}\mu^{4-D}\phi^4 - \frac{1}{16} \frac{\lambda^2}{(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta)\phi^4 \\
&+ \text{high order terms in } s.
\end{aligned} \tag{50}$$

In the effective potential all the powers ϕ^{2s} of the field will appear as stated in eq.(38). For instance, the term corresponding to the $2s - th$ power of the field is proportional to

$$\sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-s} K_{\frac{D}{2}-s}(mn\beta)\phi^{2s}. \tag{51}$$

The previous results can be used to demonstrate a first order phase transition in the $(\lambda\varphi^4)_{D=3}$ model. To simplify our discussion let us assume that is possible to truncate the series of the effective potential in $s = 3$. These does not imply the assumption that high order powers of the field gives vanishing contributions. They are simply neglected as compared to the leading terms, since we are interested in the profile of the effective potential near the origin. The coefficient of φ^6 is positive (one requires this to ensure that the truncated effective potential is bounded from below). For the sake of simplicity, let us also assume that the coefficient of the φ^6 is constant and given by σ for both cases $D = 3$ and $D = 4$. In these cases the leading contributions to the effective potential are respectively:

$$\begin{aligned}
v(\beta, \phi) &= \left(\frac{1}{2}m^2 + \frac{\lambda}{4(2\pi)^{\frac{3}{2}}} \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n}\right)^{\frac{1}{2}} K_{\frac{1}{2}}(mn\beta)\right)\phi^2 \\
&+ \left(\frac{\lambda}{4!} - \frac{\lambda^2}{16(2\pi)^{\frac{3}{2}}} \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n}\right)^{-\frac{1}{2}} K_{\frac{1}{2}}(mn\beta)\right)\phi^4 + \sigma\phi^6,
\end{aligned} \tag{52}$$

and

$$\begin{aligned}
v(\beta, \phi) = & \left(\frac{1}{2}m^2 + \frac{\lambda}{16\pi^2} \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2 \beta n} \right) K_1(mn\beta) \right) \phi^2 \\
& + \left(\frac{\lambda}{4!} - \frac{\lambda^2}{64\pi^2} \sum_{n=1}^{\infty} K_0(mn\beta) \right) \phi^4 + \sigma \phi^6.
\end{aligned} \tag{53}$$

From the above discussion, for $D < 4$ we obtain the following profile for the effective potential in the neighborhood of the origin. Below the temperature β_\star^{-1} , the dimensionless effective potential has only one global minimum. Heating the system above the temperature β_\star^{-1} , the renormalized coupling constant would become negative and the system can develop a first order phase transition since the expectation value of the order parameter changes discontinuously by temperature effects. The situation is similar to the Coleman-Weinberg mechanism for massless fields. The effects of the quantum corrections is towards the direction of breaking a symmetry. Note the similarity with the tricritical phenomena where in the tree level ($V(\varphi) = m^2\varphi^2 + \lambda\varphi^4 + \sigma\varphi^6$) the model develop a first order phase transition if we allow the coefficient of the quartic term to be negative [25].

In a detailed study, using the ring-improved one-loop effective potential, Arnold and Spinoso [26] showed that even for temperature independent coupling constant, the $\lambda\varphi^4$ model can develop at the first sight a first order phase transition. Nevertheless, these authors verified that the contribution of higher loop corrections dominates over the one-loop ring improved contributions. By these reasons, in this approximation they cannot distinguish between a first or a second order phase transition. As we discussed in the introduction, the thermal correction to the coupling constant if we include high order loops in the effective potential is positive for high temperatures.

Nevertheless for low temperatures the effective renormalized coupling constant may become negative. In this case we still have a first order phase transition. From the above discussion, we have obtained the following result: in the massive $\lambda\varphi^4$ for $D < 4$ for temperatures above β_\star^{-1} the effective potential will develop a local minimum at the origin (a false vacuum) and a global one outside the origin. In this case the initial metastable phase may decay to a stable one by nucleation of bubbles. The temperature is the parameter that drives the first order phase transition. Evaluating the ring diagrams Carrington and Takahashi independently obtained in a pure scalar model at $D = 4$ results which are consonant with our results in $D = 3$ [27].

4 The one-loop effective potential in the massless Gross-Neveu model at finite temperature.

Our purpose throughout this section is to examine the behavior of the renormalized coupling constant in a model involving fermions with a quartic interaction. In two-dimensional spacetime ($D = 2$) the model is renormalizable and ultraviolet asymptotically free. We will consider an N -component fermion field where the limit of large N will be investigated. As it was discussed in ref.(4), due to the quartic nature of the interaction, it is possible to introduce an ultralocal auxiliary scalar field φ which is formally equal to $g\bar{\psi}\psi$ where $\psi(x)$ is the fermionic field, in order to present the effective potential of the model. As we did in section II, we suppose that the quantum field is in

thermal equilibrium with a reservoir at temperature β^{-1} . We will show that for $D = 2$ and $D = 4$ in the one-loop approximation the sign of the thermal correction to the renormalized coupling constant cannot be calculated. On the other hand, for $D = 3$ in the one-loop approximation the thermal correction to the renormalized coupling constant is zero.

The Lagrange density of the massless model is given by:

$$\mathcal{L}(\bar{\psi}, \psi, \varphi) = i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{1}{2}\varphi^2 - g\varphi\bar{\psi}\psi. \quad (54)$$

Defining φ_0 as the vacuum expectation value of φ , i.e. $\varphi_0 = \langle 0|\varphi|0 \rangle = \langle 0|g\bar{\psi}\psi|0 \rangle$, the leading terms in the effective potential for large N are given by the tree-level graphs plus all one-loop graphs,

$$V(\varphi) = \frac{1}{2}\varphi_0^2 - iN \sum_{s=1}^{\infty} \frac{1}{2s} (g\varphi_0)^{2s} \int \frac{d^D q}{(2\pi)^D} \frac{1}{k^{2s}}. \quad (55)$$

After a Wick rotation we identify the effective potential as the free energy of the system. At zero temperature the model has a spontaneous breakdown of the chiral symmetry where the fermions acquire mass. The symmetry is restored at finite temperature by a second order phase transition [28]. This result can be obtained by summing the series in the effective potential. Since we are interested only in the thermal behavior of the mass and coupling constant instead of repeat the well known calculations we will adopt a very unusual road, similar to the previous chapter, by regularizing each term of the series in the effective potential before summing up.

To introduce finite temperature effects we assume that the Grassmannian integration in the path integral goes over anti-periodic configurations in Euclidean time. In the effective potential

this is equivalent to the replacement given by eq.(21) and

$$\omega \rightarrow \frac{2\pi}{\beta}(n + \frac{1}{2}). \quad (56)$$

Using eq.(33) and defining $f(D, s)$ by:

$$p(D, s) = \frac{1}{2^{2s+1}} \frac{1}{\pi^{2s-\frac{d}{2}}} \frac{(-1)^s}{s} \frac{\Gamma(s - \frac{d}{2})}{\Gamma(s)}, \quad (57)$$

it is not difficult to show that $V(\beta, \varphi_0)$ is given by:

$$V(\beta, \varphi_0) = \frac{1}{2}\varphi_0^2 + N \sum_{s=1}^{\infty} p(D, s) (g\varphi_0)^{2s} \beta^{2s-D} \sum_{n=-\infty}^{\infty} \frac{1}{(n + \frac{1}{2})^{2s-d}}. \quad (58)$$

Note that we are using dimensional regularization in eq.(55) and it is well known that for massless fields this technique requires modification in order to deal with infrared divergences [29]. Since we are regularizing only a $d = D - 1$ dimensional integral, this procedure is equivalent to inserting a mass into the d dimensional integral. In other words, the Matsubara frequency plays the role of a "mass" in the integral, provided we exclude the limit $\beta \rightarrow \infty$, which means that we must restrict ourselves to non-zero temperature.

Again, as in eq.(30), we can define a new field $\phi = \frac{\varphi_0}{\mu}$ (no confusion must be done between the present auxiliar scalar field and the previous scalar field). Using eq.(24) we obtain

$$V(\beta, \phi) = \frac{1}{2}\mu^2\phi^2 + N\mu^D \sum_{s=1}^{\infty} p(D, s) a^{\frac{D}{2}-s} (g\phi)^{2s} \sum_{n=-\infty}^{\infty} \frac{1}{(n + \frac{1}{2})^{2s-d}}. \quad (59)$$

The Hurwitz zeta function is defined as

$$\zeta(z, q) = \sum_{n=0}^{\infty} \frac{1}{(n + q)^z} \quad (60)$$

for $Re(z) > 1$ and $q \neq 0, -1, \dots$. For $q = 1$ we recover the usual Riemann zeta function. Defining:

$$r(D, s) = p(D, s) \left(\zeta(2s - d, \frac{1}{2}) + (-1)^{2s-d} \zeta(2s - d, -\frac{1}{2}) - \frac{1}{2^{d-2s}} \right) \quad (61)$$

the effective potential can be written as:

$$V(\beta, \phi) = \frac{1}{2} \mu^2 \phi^2 + N \mu^D \sum_{s=1}^{\infty} r(D, s) a^{\frac{D}{2}-s} (g\phi)^{2s}. \quad (62)$$

The effective potential is still badly defined and it will be regularized by the principle of analytic extension. The function $r(D, s)$ is valid in the begining in an open connected set of points, i.e. for $Re(z) > 1$. Since we are considering even non perturbative renormalizable models, let us study the cases $D = 2, 3$ and 4 . We would like to stress that even for the non perturbative renormalizable models it is possible to make qualitative predictions and we will regularize and renormalize the model in the standard way. A strong argument in favor of the study of the Gross-Neveu model is that the non-renormalizability does not appear in the leading $\frac{1}{N}$ approximation for $D = 3$.

After the analytic continuation, the effective potential requires a renormalization procedure in the points $s = 1, 2, \dots$. The renormalization condition which will fix the form of the counterterm of the pole $s = 1$ is:

$$\frac{\partial^2 V}{\partial \phi^2} \Big|_{\phi=cte} = \mu^2 \quad (63)$$

Due to infrared divergences, we must follow Coleman and Weinberg [13] and choose the renormalization point at non-zero ϕ . In order to evaluate the renormalized effective potential it is necessary to use the Hermite formula of the analytic extension for the Hurwitz zeta function given by [30]

$$\zeta(z, q) = \frac{1}{2q^z} + \frac{q^{1-z}}{z-1} + 2 \int_0^{\infty} (q^2 + y^2)^{-\frac{z}{2}} \sin(z \arctan \frac{y}{q}) \frac{1}{e^{2\pi y} - 1} dy. \quad (64)$$

It is not difficult to show that the thermal contribution to the renormalized coupling constant is,

$$\Delta g(\beta) = N\mu^{D-2} \sum_{s=1}^{\infty} r(D, s)(2s)(2s-1)g^{2s}(\beta\mu)^{2s-D}, \quad (65)$$

where it is understood that the polar terms in the summation have been subtracted remaining just the regular part of the analytic continuation. The situation is different from the massive $\lambda\varphi^4$ model, since we have the contribution of all terms of the series in s and the sign of the thermal contribution to the renormalized coupling constant cannot be easily obtained. Nevertheless, for sufficiently small g the leading term is $O(g^2)$. In this case, for $D = 3$ and using the fact that $\zeta(0, q) = \frac{1}{2} - q$, we obtain that $\Delta g = 0$. We found here that there is no thermal correction to the coupling constant at least in the one-loop approximation. Note that $\Delta g(\beta)$ is still not well behaved. The terms $s > \frac{D}{2}$ are divergent in the low temperature limit (the use of dimensional regularization in the beginning of the calculations leads to this situation). For $s < \frac{D}{2}$, the high temperature limit of the model is problematic due to the well known fact that ultraviolet divergences are worst as the spacetime dimension increases.

5 Conclusions

In this paper we studied the renormalization program assuming that scalar or fermionic fields are in equilibrium with a thermal reservoir at temperature β^{-1} . We have attempted to analyze the consequences of the fact that not only the renormalized mass, but also the renormalized coupling constant acquire thermal corrections.

It is well known that if we have a one spatial dimension compactified system at a finite temperature, which has a spontaneous symmetry breaking there are two different ways to restore the symmetry. Since the compactification of one spatial dimension gives us the well known mechanism of topological generation of mass, it is possible to restore the symmetry by thermal or topological effect. There is a very simple way to interpret the origin of the thermal and topological mass and coupling constant. The effective potential is not well defined. Using the Principle of the Analytic Extension, we regularize the model and the introduction of counterterms remove the principal part of the analytic extension, and the model becomes finite. Meanwhile, in the neighbourhood of the poles, the regular part of the analytic extension does not vanish. These temperature dependent regular part around the poles $s = 1$ and $s = 2$ (for $D = 4$) are identified with the thermal correction to the mass and coupling constant.

It was proved that in the $\lambda\varphi^4$ model, in the one-loop approximation, the thermal correction to the renormalized mass is positive and the thermal correction to the renormalized coupling constant is negative. In this case the renormalized coupling constant attains its maximum at zero temperature and decreases monotonically as the temperature increases. Since in $D = 4$, $\Delta\lambda(\beta)$ is $O(\lambda^2)$ it is not possible to vanish the renormalized coupling constant at a finite temperature of the thermal bath. For strong couplings ($D < 4$) there is a finite temperature where this can be achieved. For temperatures $\beta^{-1} > \beta_\star^{-1}$ (negative coupling constant) the system can develop a first order phase transition, where the origin is a false vacuum.

It is not all clear for us if at $D = 4$ the system can develop a first order phase transition. We are

using the following argument to disregard such possibility. As we discussed in the introduction, in the two-loops approximation at high temperatures the thermal correction to the coupling constant is positive. The fact that in $D = 4$ the model has a small zero temperature coupling constant eliminate the first order phase transition in $D = 4$.

We would like to emphasize that the massive $\lambda\varphi^4$ model does not belong to the same universality class of the Ising model. It is well known that it is possible to compare the $\lambda\varphi^4$ model in continuous D -dimensional Euclidean space with the Ising model. One lattice formulation can be done and the continuum limit of the model ($a \rightarrow 0$, where a is the lattice spacing) exist if the correlation length goes to infinite. This fact implies that at the continuum limit of the lattice model the system must suffer a second order phase transition. In other words, close to the critical temperature a D -dimensional Ising model has the same correlation functions as those for a field theory ($\lambda\varphi^4$ model) defined in a D -dimensional Euclidean space near the critical temperature. Since in the paper we assume that the tree level mass squared is always positive and we found that the thermal mass squared is also positive, we are always far from the critical temperature. By these reasons the system cannot fall into the universality class of a Ising model.

The analysis of this paper suggest two possible directions. First, we have to study the decay of the metastable ground state in the $(\lambda\varphi^4)_{D<4}$ model evaluating the nucleation rate per unit volume in the system. The theory of bubbles nucleation at zero and finite temperature was proposed and developed by many authors [31]. The basic result is that the probability per unit volume per unit time of the metastable vacuum to decay is given by $\Gamma = Ae^{-S(\varphi)}$, where $S(\varphi)$ is the

Euclidean action of the "bounce" solution which describes tunneling between a metastable and a true vacuum. Another possible direction is to examine if the metastability of the system (the false ground state) can be eliminated in a more general scalar model. This former subject will be presented soon in a forthcoming paper[32]. We conclude the paper with some questions which remain to be answered.

(i) Is the existence of the first order phase transition in $(\lambda\varphi^4)_{D=3}$ an artifact of our approximation? It will be interesting to obtain a non-perturbative argument to demonstrate or disprove this fact in a general way.

(i) Is the series given by eq.(69) Borel summable ? It is well known that the lack of Borel summability means that the system is unstable, since the vacuum to vacuum amplitude develop and imaginary part [33]. It would be interesting to investigate these questions.

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References

- [1] S.J.Chang, Phys.Rev.D **12**, 1671 (1975), ibid **13**, 2278 (1976), S.F.Magruder, Phys.Rev.D **14**,1602 (1976), P.Ginsparg, Nucl.Phys. **B170**, 388 (1980).
- [2] L.H.Ford and N.F.Svaiter, Phys.Rev.D **51**, 6981 (1995).
- [3] E.Speer, J.Math.Phys. **9**, 1404 (1968), C.G.Bollini, J.J.Giambiagi and A.G.Domingues, Il Nuovo Cim. **31**, 550 (1964).
- [4] G.'t.Hooft, Nucl.Phys. **B61**, 455 (1973).
- [5] E.Elizalde and K.Kirsten, J.Math. Phys. **35**, 1260 (1994), C.Villarreal, Phys.Rev.D, **51**, 2959 (1995).
- [6] D.Gross and A.Neveu, Phys.Rev.D **10**, 3235 (1974).
- [7] G.Parisi, Jour. Stat. Phys. **23**, 49 (1980).
- [8] E.Witten, Nucl. Phys. **23**, 477 (1981), M.Sher, Phys.Rev.D **24**, 1699 (1981).
- [9] H.A.Weldon, Phys.Lett. **174B**, 427 (1986).
- [10] K.Babu Joseph, V.C.Kuriakose and M.Sabir, Phys Lett **115B**, 120 (1982), O.J.P.Eboli and G.C.Marques, Phys. Lett. **162B**, 189 (1985), K.Funakubo and M.Sakamoto, Phys.Lett. **186B**, 205 (1987), P.Fedley, Phys. Lett. **196B**, 175 (1987), G.Barton, Ann of Phys. **200**, 271 (1990), M.Loewe and J.C.Rojas, Phys.Rev.D **46** 2689 (1992).

- [11] Y.Fujimoto, K.Ideura, Y.Nakano and H.Yoneyama, Phys.Lett. **167B**,406 (1986).
H.Matsumoto, Y.Nakano and H.Umezawa, Phys.Rev.D **29**, 1116 (1984).
- [12] G.Jona Lasinio, Nuov. Cim. **34**, 1790 (1964).
- [13] S.Coleman and E.Weinberg, Phys. Rev.D **7**, 1888 (1973).
- [14] P.Epstein, Math. Ann.**56**, 615 (1902).
- [15] J.F.Ashmore Nuovo Cim.Lett. **9**,289 (1972), G.G.Bolini and J.J.Giambiagi, Nuovo Cim.B **12**,
20 (1972), G.T'Hooft and M.Veltman Nucl.Phys.B **44**, 189 (1972).
- [16] K.Kirsten, J.Math.Phys.**32**, 3008 (1991).
- [17] Handbook of Mathematical Functions, edited by M.Abramowitz and I.Stegun, Dover New
York, 1965.
- [18] M.B.Kislinger and P.D.Morley, Phys.Rev.D **13**, 2771 (1976).
- [19] D.A.Kirzhnits and A.D.Linde, Phys.Lett.**42B**, 471 (1972), S.Weinberg, Phys.Rev.D **9**, 2257
(1974), C.Dolan and R.Jackiw, Phys. Rev.D **9**, 3320 (1974), D.A.Kirzhnits and A.D.Linde,
Ann.of Phys.**101**,195 (1976), P.Arnold, Phys.Rev.D **46**, 2628 (1992) .
- [20] H.W.Braden, Phys.Rev.D **25**, 1028 (1982).
- [21] J.Frohlich, Nucl.Phys. **B200**, 281 (1982).

- [22] K.Langfeld, F.Schmuser and H.Reinhardt, Phys.Rev.D **51**, 765 (1995).
- [23] K.Symanzik, Lett.Nuovo Cim. **6**, 77 (1977), R.A.Brandt, Phys.Rev.D **14**, 3381 (1976),
R.A.Brandt, Ng W. and Y.W.Bong, Phys Rev.D **19**, 503 (1979), J.Iliopoulos, C.Itzykson
and A.Martin, Rev.Mod.Phys. **47**, 165 (1975).
- [24] G.Gallavotti and V.Rivasseau, Ann.Inst. Henry Poincare, **40**, 185 (1984).
- [25] M.Blume, V.J.Emery and R.G.Griffiths, Phys.Rev.A **4**, 1071 (1971), E.K.Riedel and
F.J.Wegner, Phys.Rev.Lett.**29**, 349 (1972), H.Hamer, Phys.Rev.B**21**, 3999 (1980),
D.Boyanyovsky and L.Masperini, Phys.Rev.D **21**, 1550 (1980).
- [26] P.Arnold and O.Spinosa, Phys.Rev.D **47**, 3546 (1993).
- [27] M.E.Carrington, Phys.Rev.D, **45**, 2933 (1992), K.Takahashi, Z.Phys.C **26**, 601 (1985).
- [28] L.Jacobs, Phys.Rev.D **10**, 3956 (1974). R.F.Dashen, S.K.Ma and R.Rajaraman, Phys.Rev.D
11, 1499 (1975), B.J.Harrington and A.Yildiz, Phys.Rev.D **11**, 779 (1975), U.Wolf, Phys.Lett.
157B, 303 (1985)
- [29] G.Leibbrandt, Rev.Mod. Phys. **47**, 849 (1975).
- [30] E.T.Whittaker and G.N.Watson, in "Course of Modern Analysis" Cambridge University
Press, Cambridge, (1978).

- [31] S.Coleman, Phys.Rev.D **15**, 2929 (1977), C.G. Callan and S.Coleman, Phys.Rev.D. **16**, 1762 (1977), E.Brezin and G.Parisi, J.Stat.Phys. **19**, 269 (1978), A.D.Linde, Nucl.Phys. **B216**, 241 (1983), E.J.Weinberg and A.Wu, Phys. Rev.D **36**, 2474 (1987).
- [32] A.P.C.Malbouisson and N.F.Svaiter, On the Vacuum Stability in the Efimov-Fradkin model at Finite Temperature”, in preparation.
- [33] G.Parisi, Phys.Lett **69B**, 329 (1977), N.N.Khuri, Phys.Rev.D **16**, 1754 (1977), ibid Phys.Lett. **82B**, 83 (1979), P.Olesen, Phys.Lett. **73B**, 327 (1978).